Syllabi for Ph.D (Mathematics) Entrance Test 2023-24

Algebra

Zassenhaus's lemma, Normal and Subnormal series. Scheiers Theorem, Composition Series. Jordan-Holder theorem. Commutators and their properties. Three subgroup lemma of P.Hall. Central series. Nilpotent groups. Upper and lower central series and their properties. Invariant(normal) and chief series. Solvable groups. Derived series.

Field theory. Prime fields. Extension fields. Algebraic and transcendental extensions. Algebraically closed field. Conjugate elements. Normal extensions. Separable and inseparable extensions. Perfect fields. Finite fields. Roots of unity. Cyclotomic Polynomial in ϕ_n (x). Primitive elements.

Automorphisms of extensions. Galois extension. Fundamental theorem of Galois theory. Solutions of polynomial equations by radicals. Insolvability of the general equation of degree 5 by radicals. Construction with ruler and campass.

Canonical Forms-Similarity of linear transformations. Invariant subspaces. Reduction to triangular forms. Nilpotent transformations. Index of nilpotency. Invariants of a nilpotent transformation. The primary decomposition theorem. Jordan blocks and Jordan forms. Rational canonical form. Generalized Jordan form over any field.

Cyclic modules. Free modules. Simple modules. Semi-simple modules. Schur's Lemma. Noetherian and Artinian modules and rings Hilbert basis theorem. Wedderburn-Artin theorem. Uniform modules, primary modules, and Noether-Lasker theorem. Smith normal form over a principal ideal domain and rank. Fundamental structure theorem for finitely generated abelian groups and its application to finitely generated Abelian groups.

Real Analysis

Sequences and series of functions, point-wise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, uniform convergence and differentiation, Weierstrass approximation theorem, Power series, uniqueness theorem for power series, Abel's theorems.

Functions of several variables, linear transformations, derivatives in an open subset of R^{η}, chain rule, partial derivatives, interchange of the order of differentiation, derivatives of higher orders, Taylor's theorem, Inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method.

Definition and existence of Riemann-Stieltjes integral, properties of the integral, integration and differentiation, the fundamental theorem of Calculus, integration of vector-valued functions, rectifiable curves.

Set functions, intuitive idea of measure, elementary properties of measure, measurable sets and their fundamental properties, Lebesgue measure of sets of real numbers, algebra of measurable sets, Borel sets, equivalent formulation of measurable sets in terms of open, closed, F_σ and G_δ sets, non measurable sets.

Mechanics

Moments and products of Inertia, Theorems of parallel and perpendicular axes, principal axes, The momental ellipsoid, Equimomental systems, Coplanar distributions. Generalized cooordinates. Holonomic and Non-holonomic systems. Scleronomic and Rheonomic systems. Lagrange's equations for a holonomic system., Lagrange's equations for a conservative and impulsive forces. Kinetic energy as quadratic function of velocities. Generalized potential, Energy equation for conservative fields.

Hamilton's variables. Donkin's theorem. Hamilton canonical equations. Cyclic coordinates. Routh's equations. Poisson's Bracket. Poisson's Identity. Jacobi-Poisson Theorem. Hamilton's Principle. Principle of least action. Poincare Cartan Integral invariant. Whittaker's equations. Jacobi's equations. Statement of Lee Hwa Chung's theorem. Hamilton-Jacobi equation. Jacobi theorem. Method of separation of variables. Lagrange Brackets. Condition of canonical character of a transformation in terms of Lagrange brackets and Poisson brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations.

Gravitation: Attraction and potential of rod, disc, spherical shells and sphere. Laplace and Poisson equations. Work done by self-attracting systems. Distributions for a given potential. Equipotential surfaces. Surface and solid harmonics. Surface density in terms of surface harmonics.

Ordinary Differential Equations

Initial-value problem and the equivalent integral Equation, ε-approximate solution, Cauchy-Euler construction of an ε-approximate solution, Equicontinuous family of functions, Ascoli-Arzela theorem, Cauchy-Peano existence theorem.

Uniqueness of solutions, Lipschitz condition, Picard-Lindelof theorem for local existence and uniqueness of solutions, solution of initial-value problems by Picard method, Approximate methods of Solving first-order Equations: Power Series Methods, Numerical Methods. Continuation of Solutions, Maximum interval of existence, Extension Theorem, Dependence of solutions on initial conditions and function. Matrix method for homogeneous first order systems, nth order equation.

Total Differential Equations: Condition of Integrability, Methods of Solution. Gronwall's differential inequality, comparison theorems involving differential inequalities, zeros of solutions, Sturms separation and comparison theorems. Oscillatory and nonoscillatory equations, Riccati's Equation, Pruffer tmasformation, Lagrange's identity and Green's formula for second-order equation, Sturm-Liouville boundary-value problems, properties of eigen values and eigen functions.

Linear systems, fundamental set and fundamental matrix of a homogeneous system, Wronskian of a system. Method of variation of constants for a non-homogeneous system, reduction of the order of a homogeneous system, systems with constant coefficients, adjoint systems, periodic solutions, Floquet theory for periodic systems.

Nonlinear differential equations, plane autonomous systems and their critical points, classification of critical points-rotation points, foci, nodes, saddle points. Stability, asymptotical stability and unstability of critical points, almost linear systems, Perturbations, Simple Critical points, dependence on a parameter, Liapunov function, Liapunov's method to determine stability for nonlinear systems, limit cycles, Bendixson non-existence theorem, Statement of Poincare-Bendixson theorem, index of a critical point.

Motivating problems of calculus of variations, shortest distance, minimum surface of revolution, Brachistochrone problem, isoperimetric problem, geodesic, fundamental lemma of calculus of variations, Euler's equation for one dependent function and its generalization to 'n' dependent functions and to higher order derivatives, conditional extremum under geometric constraints and under integral constraints.

Complex Analysis

Cauchy Riemann Equations, Analytic functions, Reflection principle, Complex Integration, Antiderivatives, Cauchy-Goursat Theorem, Simply and Multiply connected domains, Cauchy's Integral formula, Higher Order derivatives, Morera's theorem, Cauchy's inequality, Liouville's theorem, The fundamental theorem of Algebra, Maximum Modulus Principle, Schwarz lemma, Poisson's formula.

Branches of many valued functions with special reference to arg z, Log z, z^a. Bilinear transformations, their properties and classification, definition and examples of conformal mapping.

Taylor's Series, Laurent's Series, Isolated Singularities, Meromorphic functions, Argument principle, Rouche's theorem, Residues, Cauchy's residue theorem, Evaluation of Integrals, Mittag Leffler's expansion theorem.

Spaces of Analytic functions, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem, Weierstrass' factorisation theorem, Gamma function and its properties, Riemann Zeta function, Riemann's functional equation. Runge's theorem.

Analytic Continuation, Uniqueness of direct analytic continuation, Uniqueness of analytic continuation along a curve, power series method of analytic continuation. Monodromy theorem and its consequences, Harmonic function on a disk, Harnack's inequality and theorem, Dirichlet problem. Green's function.

Canonical products, Jensen's formula. Poisson-Jensen formula. Hadamard's three circles theorem. Order of an entire function. Exponent of Convergence. Borel's theorem. Hadamard's factorization theorem. The range of an analytic function. Bloch's theorem. The Little Picard theorem. Schottky's theorem. Montel Caratheodory and the Great picard theorem. Univalent functions. Bieberbach's conjecture (Statement only) and the 1/4 theorem.

Measure & Integration Theory

Measurable functions and their equivalent formulations, Properties of measurable functions. Approximation of measurable functions by sequences of simple functions, Measurable functions as nearly continuous functions, Egoroffs theorem, Lusin's theorem, Convergence in measure and F. Riesz theorem for convergence in measure, Almost uniform convergence.

Shortcomings of Riemann Integral. Lebesque Integral of a bounded function over a set of finite measure and its properties, Lebesgue integral as a generalization of Riemann integral, Bounded convergence theorem, Lebesgue theorem regarding points of discontinuities of Riemann integrable functions, Integral of non-negative functions, Fatou's Lemma, Monotone convergence theorem, General Lebesgue Integral, Lebesgue convergence theorem.

Vitali's covering Lemma, Differentiation of monotonic functions, Functions of bounded variation and its representation as difference of monotonic functions. Differentiation of Indefinite .integral. Fundamental Theorem of Calculus. Absolutely continuous functions and their properties.

Topology

Definition and examples of topological spaces. Closed sets. Closure. Dense subsets. Neighbourhoods. Interior, exterior and boundary points of a set. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology. Alternate methods of defining a topology in terms of Kuratowski Closure Operator and Neighbourhood Systems. Continuous functions and homeomorphism. Connected spaces. Connectedness on the real line. Components. Locally connected spaces.

Compactness. Continuous functions and compact sets. Basic properties of compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification. Stone-Cech compactification Compactness in metric spaces. Equivalence of compactness, countable compactness and sequential compactness in metric spaces.

First and Second Countable spaces. Lindelof 's theorem. Separable spaces. Second Countability and Separability. Separation axioms. T_0, T_1, and T_2 spaces. Their characterization and basic properties. Regular and normal spaces. Urysohn's Lemma and Tietze Extension theorem. T_3 and T_4 spaces. Complete regularity and Complete normality. $T_{3/½}$ and T_5 spaces. Product topological spaces, Projection mapping. Tychonoff product topology in terms of standard sub-base and its characterizations.

Partial Differential Equations

Solution of Partial Differential Equations
Transport Equation-Initial value Problem. Non-homogeneous Equation.
Laplace's Equation-Fundamental Solution, Mean Value Formulas, Properties of Harmonic Functions, Green's Function, Energy Methods.
Heat Equation-Fundamental Solution, Mean Value Formula, Properties of Solutions, Energy Methods.
Wave Equation-Solution by Spherical Means, Non-homogeneous Equations, Energy Methods.
Nonlinear First Order PDE-Complete Integrals, Envelopes, Characteristics, Hamilton-Jacobi Equations, Hamilton's ODE, Hopf-Lax Formula, Weak Solutions, Uniqueness.
Representation of Solutions-Separation of Variables, Similarity Solutions (Plane and Travelling Waves, Solitons, Similarity under Scaling), Fourier and Laplace Transform, Hopf-Cole Transofrm, Hodograph and Legendre Transforms, Potential Functions.

Functional Analysis

Normed linear spaces, metric on normed linear spaces, Holder's and Minkowski's inequality, completeness of quotient spaces of normed linear spaces. Completeness of l_P, L^P, R^η, C^η and C[a, b]. Bounded linear transformation. Equivalent formulation of continuity. Spaces of bounded linear transformation. Continuous linear functional, conjugate spaces, Hahn Banach extension theorem (Real and Complex form). Riesz Representation theorem for bounded linear functionals on L^P and C[a, b].

Second Conjugate spaces, Reflexive spaces, uniform boundedness principle and its consequence, open mapping theorem and its application, projections, closed graph theorem, Equivalent norms, weak and strong convergence, their equivalence in finite dimensional spaces. Compact operators and its relation with continuous operators, compactness of linear transformation on a finite dimensional space, properties of compact operators, compactness of the limit of the sequence of compact operators.

Inner product spaces, Hilbert spaces, Schwarz's inequality, Hilbert space as normed linear space, convex sets in Hilbert spaces. Projection theorem, orthonormal sets, Bessell's inequality, Parseval's identity, Conjugate of a Hilbert space.